LITERATURE CITED

1. Lord Rayleigh, The Theory of Sound, Vol. 2, McGraw–Hill, New York (1948).
2. N. Rott, "The influence of heat conduction on acoustic streaming," J. Appl. Math. Phys., No. 25, 419 (1974).
3. H. Schlichting, Boundary-Layer Theory, 6th ed., McGraw-Hill, New York (1968).
4. E. D. Sorokodum and V. I. Timoshenko, "Acoustic streaming about an elliptical cylinder at large Reynolds numbers," Akust. Zh., 19, No. 6, 923 (1973).
5. W. L. Nyborg, "Acoustic streaming," in: Physical Acoustics (edited by W. P. Mason), Vol. 2B, Academic Press, New York (1965), p. 265.
6. W. L. Nyborg, "Acoustic streaming near a boundary," J. Acoust. Soc. Amer., 30, No. 4, 329 (1958).
7. L. K. Zarembo, "Acoustic streaming," in: High-Intensity Ultrasonic Fields (edited by L. D. Rozenberg), Plenum Press, New York–London (1971), p. 137.
8. B. P. Konstantinov, Hydrodynamic Sound Generation and Sound Propagation in Bounded Media [in Russian], Nauka, Leningrad (1974).
9. M. E. Arkhangel'skii and I. N. Kanevskii, "Mechanism of the acceleration of film development for normal incidence of an ultrasonic wave on the photosensitive layer," Akust. Zh., 15, No. 2, 178 (1969).

MICROCONVECTIVE HEAT-AND MASS-TRANSFER PROCESSES IN FLUIDS WITH INTERNAL ROTATION

V. G. Bashtovoi, A. N. Vislovich, and B. É. Kashevskii

UDC 536.2:532.584:538.4

A number of experimentally observed phenomena (the magnetoviscosity effect, i.e., increase of the viscosity of a ferromagnetic suspension in a magnetic field [1], and the entrainment of a polar fluid by a nonsteady magnetic field [2-4]) can be explained on the basis of the notion of internal rotations and the associated internal friction as a mechanism of momentum transfer from the field to the medium [5-8]. In line with the expanding study of the influence of internal rotations on macroscopic fluid motion there is also considerable interest in the development of mathematical models of asymmetric polarizable and magnetizable media [5, 9-12].

In the present article we show that the influence of internal rotations under definite conditions not only leads to a modification of the momentum-transfer law, but also proves significant in heat-transfer processes and, in the case of multicomponent fluids, mass-transfer processes as well, giving rise to a highly specific "microconvective" transfer mechanism.

Inasmuch as the significance of the internal-rotation concept is particularly highlighted in the case of suspensions and colloidal solutions, we discuss a certain volume of a suspension in a system S', in which macroscopic motion does not take place. This system rotates relative to the laboratory frame S with an angular velocity $\boldsymbol{\Omega} = (1/2)\operatorname{rot}\mathbf{v}$ (rot = curl). In the system S' the particles of the suspension rotate with a velocity $\mathbf{R} = \boldsymbol{\omega} - \boldsymbol{\Omega}$, where $\boldsymbol{\omega}$ is their rotational velocity in the system S. The rotating particles together with the fluid entrained by them through viscosity induce a local microconvective heat transfer in the system S' in the case of a nonuniform temperature distribution in the fluid. When the distance between the particles is commensurate with their sizes and the latter are large, a possible outcome of the interaction of the temperature fields of the individual microvortices and heat transfer between them is a macroscopic heat flux q_r, which competes with the conductive heat flux q_0.

We estimate the ratio q_r/q_0 on the basis of the heat-transfer equation $\mathbf{v}\nabla T = \varkappa\nabla^2 T$, applying it to the individual microvortex, in which case it is necessary to adopt as the characteristic space scale the microvortex radius l_0. Then $v \simeq R l_0$, and:

Minsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 3, pp. 88-93, May-June, 1978. Original article submitted March 31, 1977.

0021-8944/78/1903-0351 $07.50

$$q_r/q_0 \simeq R l_0^2/\varkappa. \tag{1}$$

If $l_0 \simeq 100\,\mu$ and $\varkappa = 10^{-7}$ m^2/sec, then $q_r/q_0 \simeq 1$ for $R \simeq 10$ sec^{-1}.

Thus, a sizable net effect is to be expected in the case of relatively large vortices (~10 to 100 μ).

A complete theoretical solution of the heat-transfer problem in the given situation can be obtained by analyzing the hydrodynamics and total heat transfer in application to each individual microvortex on the basis of equations describing the carrier fluid with regard for interaction of the microvortex temperature and velocity fields. Since this problem appears to be unsolvable analytically, we propose here a phenomenological approach, wherein rather than consider all the details of microconvective heat transfer we introduce the concept of the effective thermal conductivity tensor λ_{ik}, which has the property that the heat flux determined by it – $q_i = \lambda_{ik}\nabla_k T$ – is equal to the flux induced by the actual mechanisms: microconvection and conduction. The tensor λ_{ik} is a function of the vector **R**. The general form of the tensor constructed from the components of **R** is $\lambda_{ik} = \lambda_1\delta_{ik} - (\lambda_r/R^2)R_iR_k + (\lambda_a/R)\varepsilon_{ikm}R_m$, where ε_{ikm} is the Levi-Civita tensor.

Inasmuch as the fluid motion induced in the microvortices by microconvective heat transfer takes place in a plane perpendicular to the vector R, it is reasonable to assume that the heat transfer along R is solely attributable to true heat conduction. This inference is embodied mathematically in the relation $(\mathbf{q}\cdot\mathbf{R}) = -\lambda_0(\nabla T\cdot\mathbf{R})$, which asserts that $\lambda_1 - \lambda_r = \lambda_0$. Consequently,

$$\lambda_{ik} = (\lambda_0 + \lambda_r)\delta_{ik} - \lambda_r e_i e_k + \lambda_a \varepsilon_{ikm} e_m, \tag{2}$$

where $e_s = R_s/R$.

The constraints on the signs of the coefficients λ_a, λ_r, λ_0 must be deduced from the condition of nonnegativity of the net entropy σ_T produced by the effective thermal conductivity [13]:

$$T^2\sigma_T = -\mathbf{q}\cdot\nabla T \geqslant 0$$

or

$$(\lambda_0 + \lambda_r)(\nabla T)^2 - \lambda_r e_i e_k \nabla_i T \nabla_k T \geqslant 0. \tag{3}$$

This result is valid for any values of e_m, specifically if $e_i = 0$. Therefore, $\lambda_0 + \lambda_r \geq 0$. By the independence of conduction and microconvection we obtain

$$\lambda_0,\ \lambda_r \geqslant 0. \tag{4}$$

Relation (4) automatically ensures satisfaction of Eq. (3), because $\lambda_r(\nabla T)^2 \geq \lambda_r e_i e_k \nabla_i T \nabla_k T$. The sign of λ_a remains indeterminate, because the heat flux due to the antisymmetric part of the thermal conductivity tensor $\lambda_a \varepsilon_{ikm} e_m$, yields a zero contribution to the net entropy: $\varepsilon_{ikm} e_m \nabla_k T \nabla_i T \equiv 0$.

The coefficients λ_r and λ_a depend on the thermophysical characteristics of the carrier fluid and on the sizes and concentration of the particles, and for a given medium they are functions only of the scalar invariant of R, i.e., $|\mathbf{R}|$.

It is important to note that the change of the tensorial dimensions of the transfer coefficients for shear flows of fluids characterized by internal structure is discussed in [14], and the tensorial nature of the thermal conductivity of a medium in the presence of a vector field of internal rotations is described from the formal standpoint in [10].

We now write in vector form the expression for the heat flux in a medium whose thermal conductivity is described by relation (2):

$$\mathbf{q} = -\lambda_0\nabla T - \lambda_r(\nabla T - \mathbf{e}(\mathbf{e}\cdot\nabla T)) - \lambda_a \mathbf{e}\times\nabla T. \tag{5}$$

The first term in this expression corresponds to the true thermal conductivity of the medium, the second term specifies the microconvective heat flux in a plane perpendicular to the vector e and parallel to the projection of the temperature gradient onto that plane, and the third term describes, correct to the sign of λ_a, the microconvective heat transfer in the direction perpendicular to the vectors e and ∇T. The diffusion tensor D_{ik} of the component dissolved in the medium with microrotations has a form similar to that of the thermal conductivity tensor (2).

For the thermomechanical description of a medium with the transfer properties discussed above we must resort to the formal machinery of asymmetrical hydrodynamics. Neglecting diffusion of internal rotations, compressibility, and dissipative heat-release processes, we write the transfer equations for an asymmetrical fluid in the form:

$$\rho dv_i/dt = \partial\sigma_{ik}/\partial x_k + \rho f_i; \tag{6}$$

$$Id\omega_i/dt = -\varepsilon_{imn}\,(1/2)(\sigma_{mn} - \sigma_{nm}) + \rho m_i; \quad c_p\rho dT/dt = -\mathrm{div}\,\mathbf{q};\ \mathrm{div}\,\mathbf{v} = 0, \tag{7}$$

where I is the total moment of inertia of the particles per unit volume and f_i and m_i are the densities of the external forces per unit mass and the moments of the forces.

The presence of external volume torques is a necessary condition for the production of internal rotations in the fluid. Consequently, for experimental observation of the microconvective heat-transfer effect one can use ferromagnetic suspensions. Internal rotations can be created in such suspensions either by a rotating field or by uniform magnetic field applied to a shear flow of the suspensions. We consider the second technique, which appears to be the more practical to implement.

It is essential to note that the thermal conductivity of a ferromagnetic suspension in the presence of a magnetic field is also a function of the field vector **H**: $\lambda_{ik}=\lambda_{ik}(\mathbf{R}, \mathbf{H})$. Inasmuch as the field-induced anisotropy of the thermal conductivity is associated with the structure imparted to the suspension by magnetic dipole−dipole forces, and as the internal-rotation-induced anisotropy is associated with structural disintegrations, the influence of the field on the thermal conductivity of the medium can be neglected in the first stage of investigation of microconvective heat transfer.

We invoke the expression obtained by Kagan and others [4] for the stress tensor of an incompressible ferromagnetic suspension in the near-equilibrium approximation with respect to magnetization (it is assumed that the characteristic hydrodynamic time constants are small in comparison with the magnetization relaxation time of the suspension):

$$\sigma_{ik} = p\delta_{ik} + \eta\left(\frac{\partial v_i}{\partial x_k} + \frac{\partial v_k}{\partial x_i}\right) + \frac{H_i B_k}{4\pi} - 2\eta_r\varepsilon_{ikl}\,(\Omega_l - h_l\,(\mathbf{\Omega}\cdot\mathbf{h})). \tag{8}$$

Here η and η_r are the ordinary and rotational viscosities, respectively. The value of η_r for a given suspension is a function of the field H. Without writing out the explicit form of η_r calculated in [4] with stringent constraints on the properties of the suspension, we assume that relation (8) is applicable to magnetic suspensions over a wide range of their dispersion and other characteristics. Also, $h_l=H_l/H$; $B_i=H_i+4\pi M_i$ is the magnetic induction, and M_i is the magnetization of the suspension. The steady-state equations of motion of a ferromagnetic suspension in uniform fields, according to (6)-(8), have the form

$$\rho(\mathbf{v}\nabla)\mathbf{v} = -\nabla p + \eta\nabla^2\mathbf{v} - \eta_r\,\mathrm{rot}\,(\mathbf{\Omega} - \mathbf{h}(\mathbf{\Omega}\cdot\mathbf{h})); \tag{9}$$

$$\mathbf{R} = (\eta_r/\eta_r^0)\,\mathbf{\Omega}, \tag{10}$$

where $\eta_r^0=\max\,\eta_r$ is the value of the rotational viscosity when the rotations are completely frozen ($\omega=0$).

We augment Eqs. (9) and (10) with the steady-state heat-conduction equation

$$c_p\rho\mathbf{v}\nabla T = -\mathrm{div}\mathbf{q}, \tag{11}$$

in which **q** is given by expression (5).

Proceeding from relations (5), (9)-(11), we consider nonisothermal Couette flow in the annular space between two long cylinders when the inner cylinder of radius R_1 is rotating with a velocity Ω_0 and a constant heat flux q_0 is specified on it, while the outer cylinder of radius R_2 is at rest and is thermostatically regulated at a constant temperature T_0. The magnetic field is uniform and perpendicular to the generatrix of the cylinder.

In this situation Eq. (9) describes flow with a constant shear velocity:

$$\mathbf{\Omega} = -\frac{R_1^2\Omega_0}{R_2^2 - R_1^2}\mathbf{i}_z.$$

This fact coupled with Eq. (10) leads to the conclusion that the suspended particles experience slip relative to the carrier fluid, with an angular velocity

$$\mathbf{R} = -\frac{\eta_r R_1^2\Omega_0}{\eta_r^0\,(R_2^2 - R_1^2)}\mathbf{i}_z. \tag{12}$$

In the given problem, since the internal rotations are uniform, they do not alter the temperature profile or the velocity profile of the fluid, rather they merely intensify the heat-transfer process. The solution of

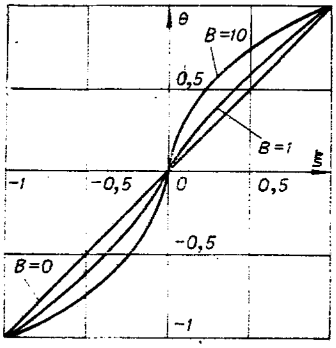

Fig. 1

the heat-conduction equation, which takes the form $\nabla^2 T = 0$ with boundary conditions $T(R_2) = T_0$, $T'(R_1) = -q_0/\lambda^*$, where $\lambda^* = \lambda_0 + \lambda_r$, is

$$T = T_0 + (q_0 R_1/\lambda^*) \ln (R_2/r).$$

The temperature T_1 on the inner cylinder is

$$T_1 = T_0 + (q_0 R_1/\lambda^*) \ln (R_2/R_1). \tag{13}$$

If the ratio η_r/η_r^0 is known, then by determining T_1, q_0, and Ω_0 experimentally it is possible to determine the function $\lambda_r(R)$ according to (13) and (12) for a given suspension, such being the objective of the experiment. A better technique for determining η_r/η_r^0 is to measure η_r together with T_1, q_0, and Ω_0 in a single experiment according to the value of the torque acting on one of the cylinders.

In another situation that is also simple to realize experimentally, namely, nonisothermal Poiseuille flow in a narrow gap between two horizontal thermostatically regulated plates in a uniform perpendicular field, not only the heat flux, but also the temperature profile of the liquid changes. This happens because the internal rotations and, hence, the thermal conductivity in the layer vary across the layer from one point to the next. In the given situation the equation of motion (9) is solved independently of the heat-conduction equation. With the boundary conditions $v_x|_{y=h} = v_x|_{y=-h} = 0$ this solution turns out to be

$$v_x = \frac{\nabla p}{2\eta^*}(h^2 - y^2),\ v_y = v_z = 0;\ \Omega_z = -\frac{\partial v_x}{\partial y} = \frac{\nabla p}{\eta^*} y,$$

where $\eta^* = \eta_r + \eta$. The intensity of the internal rotations is given by the expression

$$R_z = (\eta_r \nabla p/\eta_r^0 \eta^*) y. \tag{14}$$

Proceeding from this result, we analyze the thermal situation in the layer, assuming that λ_r is a linear function of $|R|$:

$$\lambda_r = \alpha |R|,\ \alpha > 0. \tag{15}$$

Relations (15) and (14) give

$$\lambda_r = \beta |y|,\ \beta = \alpha \eta_r \nabla p/\eta_r^0 \eta^*.$$

Inasmuch as the derivative λ'_r is discontinuous at the point $y = 0$, the solution of the heat-conduction equation, which in this case has the form

$$(1 + \lambda_r/\lambda_0) T'' + (\lambda'_r/\lambda_0) T' = 0,$$

must be sought separately in the domain $0 < y \le h (\lambda'_r = \beta)$ and in the domain $-h \le y < (\lambda'_r = -\beta)$, and then the solutions matched at $y = 0$.

Finally, introducing the dimensionless variables $\xi = y/h$, $\Theta = [2T - (T_1 + T_2)]/(T_1 - T_2)$ with the boundary conditions $\Theta(-1) = -1$, $\Theta(1) = 1$, we obtain

$$\Theta = \operatorname{sgn}(\xi) \ln (1 + B|\xi|)/\ln (1 + B), \tag{16}$$

where $B = \beta h/\lambda_0$ is determined by the ratio of the maximum value of the effective thermal conductivity coefficient λ_r associated with internal rotations to the thermal conductivity λ_0 of the rotationless fluid and characterizes the nonlinearity of the temperature profile. As $B \to 0$ (the influence of internal rotations diminishing), expression (16) goes over to $\Theta = \xi$, i.e., gives the usual linear temperature distribution along the height of the layer.

The change in behavior of the temperature distribution of the liquid in the layer with increasing value of the parameters B is illustrated in Fig. 1.

It is seen that as B is increased the temperature profile becomes increasingly curved in such a way that the thermal stresses, decreasing near the plate, extend into the central region of the layer, where $\partial\Theta/\partial\xi = B/\ln(1 + B)$.

It is also seen that the increase in the heat flux q through the layer due to internal rotations is given by the expression

$$q/q_0 = B/\ln(1 + B),$$

which in the case of small values of B goes over to the relation $q = q_0(1 + B/2)$. For experimental observation of the effect in question it appears more practical to measure the temperature drop between the boundaries of a horizontal layer, in which case the lower plate is thermostatically regulated at $T = T_0$ and a constant heat flux q is maintained at the upper plate.

Reducing the temperature to dimensionless form by reference to the temperature drop that occurs in the absence of internal rotations (H = 0): $\Theta = (T - T_0)/(\hat{T}_1 - T_0)$, where $\hat{T}_1$ is the temperature of the upper plate in the absence of rotations and $\hat{T}_1 - T_0 = 2qh/\lambda_0$, and using the relation $y = \xi h$, we find a solution of the stated problem in the form

$$\Theta_1 = (T_1 - T_0)/(\hat{T}_1 - T_0) = \ln(1 + B)/B. \tag{17}$$

We now obtain a simple estimate of the reduction of the thermal stress in the layer for a given heat flux, assuming in accordance with (1) that $B = Rl_0^2/\varkappa$ and, in accordance with (14) that $R = \eta_r \nabla ph/\eta_r^0 \eta^*$. Putting $\eta_r/\eta_r^0 = 1$, $\eta^* = 10^{-2}$, $\nabla p = 0{,}6$, $h = 1$, $l_0 = 10^{-2}$, $\varkappa = 10^{-5}$ (cgs units), we obtain $B \simeq 6$. Substituting the value found for B into (17), we find that the temperature drop decreases by one third ($\Theta_1 = 0.3$).

Thus, the mechanism discussed here can result in appreciable intensification of heat transfer and modification of the temperature profile in shear flows of suspensions in which volume torques are present.

LITERATURE CITED

1. J. P. McTaque, "Magnetoviscosity of magnetic colloids," J. Chem. Phys., 51, 133 (1969).
2. V. Zwetkoff, "Bewegung anisotroper Flussigkeiten im rotierenden Magnetfeld," Acta Physicochim., 10, No. 4, 554 (1939).
3. R. Moskowitz and R. E. Rosensweig, "Nonmechanical torque-driven flow of a ferromagnetic fluid by an electromagnetic field," Appl. Phys. Lett., 11, No. 10, 301 (1967).
4. I. Ya. Kagan, V. G. Rykov, and E. I. Litovskii, "Flow of a dielectric ferromagnetic suspension in a rotating magnetic field," Magnitn. Gidrodinam., No. 2, 135 (1973).
5. M. I. Shliomis, "Effective viscosity of magnetic suspensions," Zh. Éksp. Teor. Fiz., 61, No. 6(12), 2411 (1971).
6. Yu. A. Glazgov, "Role of higher harmonics in the motion of ferrosuspensions in a rotating magnetic field," Magnitn. Gidrodinam., No. 4, 31 (1975).
7. A. O. Tsebers, "Interphase stresses in the hydrodynamics of fluids with internal rotation," Magnitn. Gidrodinam., No. 1, 79 (1975).
8. A. N. Vislovich, "Action of a rotating field on a ferromagnetic suspension in a layer with a free boundary," Pis'ma Zh. Éksp. Teor. Fiz., 1, No. 16, 744 (1975).
9. J. T. Jenkins, "A theory of magnetic fluids," Arch. Rat. Mech. Anal., 46, No. 1, 42 (1972).
10. V. M. Suyazov, "Structural-continuum approach to the magneto- and electrorheology of disperse systems," Magnitn. Gidrodinam., No. 2, 3 (1972).
11. A. O. Tsbers, "Flow of dipole fluids in external fields," Magnitn. Gidrodinam., No. 4, 3 (1974).
12. V. G. Bashtovoi and B. É. Kashevskii, "Asymmetrical model of a magnetic fluid with allowance for finite anisotropy of the ferromagnetic particles," Magnitn. Gidrodinam., No. 4, 24 (1976).
13. S. R. de Groot and P. Mazur, Non-Equilibrium Thermodynamics, North-Holland Publ. Co., Amsterdam; Wiley, New York (1964).
14. A. V. Lykov and B. M. Berkovskii, "Transfer laws in non-Newtonian fluids," in: Heat and Mass Transfer in Non-Newtonian Fluids [in Russian], Énergiya, Moscow (1968).